REDUCED FORM BARTIK ROTEMBERG WEIGHT

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Let our Bartik measure be $B_i = \sum_k Z_{ik} g_k$, controls W_i (including a constant) and outcome Y_i. We're running the regression

$$Y_i = W_i \gamma + B_i \beta + \epsilon_i.$$

We do residual regression, and denote $B_i^{\perp} = M_W B_i$, $Y_i = M_W Y_i$, such that

$$Y_i^{\perp} = B_i^{\perp} \beta + \epsilon_i^{\perp}.$$

Our estimate for β can be written as (since M_W is idempotent):

$$\hat{\beta} = (B' B^{\perp})^{-1} (B' Y^{\perp})$$

.

As a result, it is possible to rewrite this as:

$$\hat{\beta} = \sum_k \hat{\alpha}_k \hat{\beta}_k, \qquad \hat{\beta}_k = (Z_k' B^{\perp})^{-1} (Z_k' Y^{\perp}), \qquad \hat{\alpha}_k = \frac{g_k Z_k' B^{\perp}}{\sum_{k'} g_{k'} Z_{k'}' B^{\perp}},$$

.

Hence, each β_k can be recovered as using the industry share as an instrument for the reduced form measure.