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STRUCTURAL EQUATION METHODS IN THE SOCIAL SCIENCES

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This survey of the use of structural equation models and methods by social scientists emphasizes the treatment of unobservable variables and attempts to redress economists' neglect of the work of Sewall Wright.

"Diuerse pathes leden diuerse folke the rihte wey to Roome."
—Geoffrey Chaucer, *A Treatise on the Astrolabe*, 1391.

1. INTRODUCTION

IN HIS 1966 Presidential Address to the Econometric Society, Herman Wold spoke on the problems of nonexperimental model building. Wold (1969, p. 372) declared that:

> . . . econometrics has played a pioneering part in posing these problems and in establishing principles and methods of general scope for their treatment. Its influence is very broad, serving as a pioneer in the wide field of the social sciences, and in the still wider realm of nonexperimental model building.

The present paper is devoted to a similar subject. By structural equation models, I refer to stochastic models in which each equation represents a causal link, rather than a mere empirical association. The models arise in nonexperimental situations and are characterized by simultaneity and/or errors in the variables. The errors in the variables may be due to measurement error in the narrow sense, or to the fact that measurable quantities are not the same as the relevant theoretical quantities. Generally speaking the structural parameters do not coincide with coefficients of regressions among observable variables, but the model does impose constraints on those regression coefficients. As a consequence, we face subtle issues of identification and draw upon elaborate methods of statistical inference.

My subject is similar to Wold's, but my message is somewhat different. Students of the American West will recall that before the pioneers arrived, explorers had already charted the territory. Then, while one group of pioneers was pitching its tents, forging tools, sowing seeds, and building model cabins, a second wagon train had followed a different trail and found *its* promised land. The explorers and their charts were sometimes forgotten and the neighboring pioneers ignored. I propose that we pause from our labors in the fertile valley of econometrics to first recall an explorer who was there before us all (but whose rough chart we

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have forgotten), and then to cross the hills and visit our neighbors. I hasten to add that we need not fold our tents and settle in greener pastures. Instead, we may return with fresh tools and new seeds. Hybridization may invigorate even the most productive species.

2. SCHULTZ AND WRIGHT

Let us begin with one of our own official pioneers. Henry Schultz (1938, pp. 72–80) considered the problem of simultaneity in his classic treatise, *The Theory and Measurement of Demand*. "Is it possible," he asked, "to deduce statistically the theoretical demand (or supply) curve when we know only ... [their] points of intersection ... at different points of time?" It is possible, he replied, provided that the curves remain fixed and that "a fixed time interval elapses between changes in price and corresponding changes in supply." In such a situation, the statistician "can determine the demand curve by relating the price ... to the quantity for the same year, and he can determine the supply curve by relating the price ... to the quantity for the following year."

In effect Schultz adopted the cobweb model,

$$q^D = \alpha p + u, \qquad q^S = \beta p_{-1} + v, \qquad q^D = q^S \ (= q), \tag{1}$$

and argued that lagged price, being predetermined, serves to identify the structural model. He gave no clues as to other means of attaining identification. The cobweb specification clearly dictates a particular estimation procedure: regress q on p_{-1} to estimate β, and regress p on q to estimate $1/\alpha$. (As Wold (1969, pp. 373–374) reminds us, " 'The choice of regression' is a choice between causal models, and the choice between the models settles the choice between the regressions.") Overlooking this, Schultz discussed the choice of dependent variable for the demand equation exclusively in terms of errors in the variables. His pragmatic solution was to compute both the price-on-quantity and the quantity-on-price regression lines, placing somewhat more weight on the latter on the grounds that the quantity series was more subject to measurement error. This agnostic procedure was followed in the analysis of demand for ten agricultural products which makes up the empirical core of his masterful book.

Actually, Schultz had arrived at more or less this point ten years earlier. In his 1928 study of the sugar market he considered identification and estimation (p. 130) as follows: "In deriving the demand curve for a given commodity, the question that we put to ourselves is 'What is the relation between the price for any given year . . . and the consumption for the same year?' The statistical procedure suggested by this question is to correlate consumption with price for identical years.... If, after allowing for the 'disturbing factors' affecting our variables, we obtain a negative correlation, suggesting that as prices rise consumption falls off, we are probably dealing with a true demand curve." Already concerned with errors in the variables, he related consumption with price in three ways: computing the q-on-p, the p-on-q, and the orthogonal regression as estimates of the demand curve. The supply situation he deemed more complicated, because "the question we try to answer when we attempt to derive a supply curve, namely 'What is the relation between changes in prices and changes in production to which they give rise?' does not suggest a safe statistical procedure, for the reason that we cannot tell a priori which are the prices that have called forth any given production, whether they are present prices, past prices, or estimated future prices." For sugar, Schultz argued that past prices are typically relevant and reported the three regressions between q and p_{-1} as estimates of the supply curve.

Clearly, this is the work of a pioneer who recognized the issues of identification and estimation which continue to concern us. Perhaps his vision was limited—the models were confined to two equations, lagged response was the only basis for identification, and the connection between specification and estimation was overlooked. But can we today object to these defects in pioneering works published in 1938 and 1928?

In 1925, the United States Department of Agriculture published a sixty page bulletin entitled *Corn and Hog Correlations.* The author's identification was also modest: "Sewall Wright, Animal Husbandman, Animal Husbandry Division, Bureau of Animal Industry." But the bulletin contains empirical estimates of the following structural model:

$$
\begin{aligned}
y_1 &= a_1 x_1 + \varepsilon_1, \\
y_2 &= g y_1 + h x_2 + i x_4 + j x_5 + \varepsilon_2, \\
y_3 &= k y_1 + l x_2 + m x_4 + \varepsilon_3, \\
y_4 &= b y_2 + a y_1 + c x_2 + e x_3 + f x_4 + d x_7 + \varepsilon_4, \\
y_5 &= e_1 y_4 + e_2 x_4 + e_3 x_5 + \varepsilon_5, \\
y_6 &= f_1 y_4 + f_2 x_4 + f_3 x_5 + f_4 x_6 + \varepsilon_6,
\end{aligned}
\tag{2}
$$

where y_1 is corn price, y_2 is summer hog price, y_3 is winter hog price, y_4 is hog breeding, y_5 is summer hog average weight, y_6 is winter hog quantity, x_1 is lagged

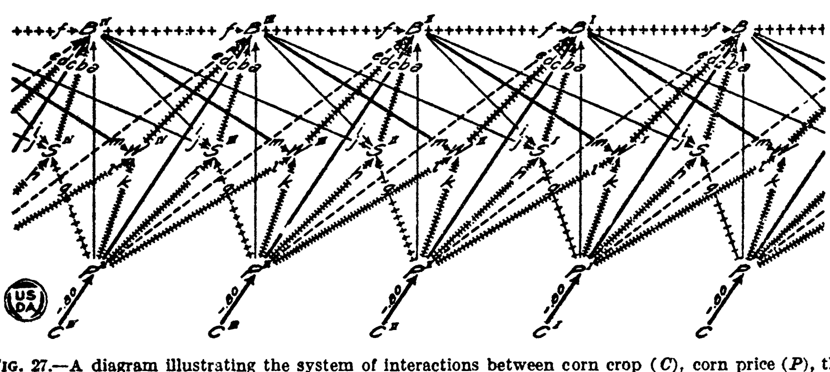


FIG. 27.—A diagram illustrating the system of interactions between corn crop (C), corn price (P), the summer price of hogs (S), the winter price of hogs (W), and the amount of hog breeding (B) in successive years, which has been found to be most successful in explaining the observed correlations. The negative paths of influence are represented by plain arrows, the positive, by crosshatched arrows. The most important paths ($c=-.85$, $d=+.65$, $m=-.65$, $p_{pc}=-.80$) are represented by heavy lines, the least important ones ($e=-.15$, $f=+.10$, $g=+.15$) by broken lines, and the paths of intermediate importance ($a=-.45$, $b=+.35$, $h=+.50$, $i=-.40$, $j=-.40$, $k=+.45$, $l=+.25$) by light lines

FIGURE 1A.—Wright (1925).

THE EQUATIONS EXPRESSING THE CORRELATIONS WITHIN THE CENTRAL SYSTEM OF VARIABLES, CORN PRICE (P), SUMMER HOG PRICE (S), WINTER HOG PRICE (W), AND HOG BREEDING (B) AS THE RESULTANT OF PATH COEFFICIENTS

Actual correlations are given in the second column, breeding being represented in parentheses by summer weight of the same year and winter pack of the following year. The calculated correlations are given in the last column. [Roman numerals denote successive lags.]

			Calculated value
r_{SP}	$= +0.18$	$= g$	$+0.15$
r_{BP}	$= (-0.43, -0.32)$	$= a + br_{SP}$	-0.40
r_{WP}	$= +0.40$	$= k$	$+0.45$
$r_{SP^{I}}$	$= +0.66$	$= h + ir_{BP}$	$+0.66$
$r_{BP^{I}}$	$= (-0.51, -0.53)$	$= br_{SP^{I}} + c + dr_{WP} + fr_{BP}$	-0.37
$r_{WP^{I}}$	$= +0.56$	$= l + mr_{BP}$	$+0.51$
$r_{SP^{II}}$	$= +0.29$	$= ir_{BP^{I}} + jr_{BP}$	$+0.31$
$r_{BP^{II}}$	$= (+0.13, -0.01)$	$= br_{SP^{II}} + dr_{WP^{I}} + e + fr_{BP^{I}}$	$+0.25$
$r_{WP^{II}}$	$= +0.20$	$= mr_{BP^{I}}$	$+0.24$
$r_{SP^{III}}$	$= -0.04$	$= ir_{BP^{II}} + jr_{BP^{I}}$	$+0.05$
$r_{BP^{III}}$	$= (+0.22)$	$= br_{SP^{III}} + dr_{WP^{II}} + fr_{BP^{II}}$	$+0.20$
$r_{WP^{III}}$	$= -0.31$	$= mr_{BP^{II}}$	-0.16
$r_{SP^{IV}}$	$=$	$= ir_{BP^{III}} + jr_{BP^{II}}$	-0.18
$r_{BP^{IV}}$	$=$	$= br_{SP^{IV}} + dr_{WP^{III}} + fr_{BP^{III}}$	-0.15
$r_{WP^{IV}}$	$=$	$= mr_{BP^{III}}$	-0.13
$r_{BB^{I}}$	$= (+0.05, +0.21)$	$= [r_{BP^{I}}(dl + e) + r_{BP}(bh + c + dk) + bi + f]/[1 - bj - dm]$	$+0.07$
$r_{SP^{I}}$	$= (-0.08, -0.50)$	$= hr_{BP} + i + jr_{BB^{I}}$	-0.63
r_{WB}	$= (-0.49, -0.45)$	$= kr_{BP} + lr_{BP^{I}} + mr_{BB^{I}}$	-0.32
$r_{WB^{I}}$	$= (-0.63, -0.58)$	$= lr_{BP} + m$	-0.75
$r_{SP^{II}}$	$= (-0.31, -0.18)$	$= ir_{BB^{I}} + j$	-0.43
$r_{WB^{II}}$	$= (+0.07, +0.20)$	$= mr_{BB^{I}}$	-0.05
$r_{BB^{II}}$	$= (-0.61, -0.36)$	$= br_{SB^{II}} + dr_{WB^{I}} + er_{BP} + fr_{BB^{I}}$	-0.57
$r_{SB^{III}}$	$= (\quad, +0.51)$	$= ir_{BB^{II}} + jr_{BB^{I}}$	$+0.20$
$r_{WB^{III}}$	$= (\quad, +0.48)$	$= mr_{BB^{II}}$	$+0.37$
$r_{BB^{III}}$	$= (\quad, -0.01)$	$= br_{SB^{III}} + dr_{WB^{II}} + fr_{BB^{II}}$	-0.02
$r_{SB^{IV}}$	$=$	$= ir_{BB^{III}} + jr_{BB^{II}}$	$+0.23$
$r_{WB^{IV}}$	$=$	$= mr_{BB^{III}}$	$+0.01$
$r_{BB^{IV}}$	$=$	$= br_{SB^{IV}} + dr_{WB^{III}} + fr_{BB^{III}}$	$+0.32$
$r_{SB^{V}}$	$=$	$= ir_{BB^{IV}} + jr_{BB^{III}}$	-0.12
$r_{WB^{V}}$	$=$	$= mr_{BB^{IV}}$	-0.21
$r_{BB^{V}}$	$=$	$= br_{SB^{V}} + dr_{WB^{IV}} + fr_{BB^{IV}}$	0.00
$r_{SB^{VI}}$	$=$	$= ir_{BB^{V}} + jr_{BB^{IV}}$	-0.13
$r_{WB^{VI}}$	$=$	$= mr_{BB^{V}}$	0.00
$r_{BB^{VI}}$	$=$	$= br_{SB^{VI}} + dr_{WB^{V}} + fr_{BB^{V}}$	-0.18
r_{WS}	$= +0.69$	$= kr_{SP} + lr_{SP^{I}} + mr_{SB^{I}}$	$+0.64$
$r_{SW^{I}}$	$= +0.76$	$= hr_{WP} + ir_{WB} + jr_{WB^{I}}$	$+0.65$
$r_{WW^{I}}$	$= +0.36$	$= lr_{WP} + mr_{WB}$	$+0.32$
r_{BS}	$= (-0.12, -0.08)$	$= ar_{SP} + b + cr_{SP^{I}} + dr_{SW^{I}} + er_{SP^{II}} + fr_{SB^{I}}$	$+0.04$
$r_{BW^{I}}$	$= (+0.33, +0.11)$	$= br_{SW^{I}} + cr_{WP} + d + er_{WP^{I}} + fr_{WB}$	$+0.39$
$r_{SS^{I}}$	$= +0.24$	$= hr_{SP} + ir_{BS} + jr_{SB^{I}}$	$+0.31$
$r_{WS^{I}}$	$= -0.09$	$= lr_{SP} + mr_{BS}$	$+0.01$
$r_{SW^{II}}$	$= -0.11$	$= ir_{BW^{I}} + jr_{WB}$	-0.03
$r_{WW^{II}}$	$= -0.32$	$= mr_{BW^{I}}$	-0.25
$r_{BS^{I}}$	$= (+0.24, +0.11)$	$= br_{SS^{I}} + cr_{SP} + dr_{WS} + er_{SP^{I}} + fr_{BS}$	$+0.30$
$r_{BW^{II}}$	$= (+0.13, \quad)$	$= br_{SW^{I}} + dr_{WW^{I}} + er_{WP} + fr_{BW^{I}}$	$+0.17$
$r_{SS^{II}}$	$= -0.26$	$= ir_{BS^{I}} + jr_{BS}$	-0.14
$r_{WS^{II}}$	$= -0.34$	$= mr_{BS^{I}}$	-0.20
$r_{SW^{III}}$	$=$	$= ir_{BW^{II}} + jr_{BW^{I}}$	-0.22
$r_{WW^{III}}$	$=$	$= mr_{BW^{II}}$	-0.11
$r_{BS^{II}}$	$= (-0.04, \quad)$	$= br_{SS^{II}} + dr_{WS^{I}} + er_{SP} + fr_{BS^{I}}$	-0.05
$r_{BW^{III}}$	$=$	$= br_{SW^{III}} + dr_{WW^{II}} + fr_{BW^{II}}$	-0.22

FIGURE 1B.—Wright (1925).

corn crop, x_2 is lagged corn price, x_3 is twice-lagged corn price, x_4 is lagged hog breeding, x_5 is twice-lagged hog breeding, x_6 is thrice-lagged hog breeding, and x_7 is lagged winter hog price. The ε's are disturbances, and the breeding variables y_4, x_4, x_5, and x_6 are unobserved.

To estimate this model of six equations in thirteen variables, Wright drew up a flow chart, Figure 1A, and read off the chart a set of equations in which zero covariances are exploited to express moments among observable variables in terms of structural parameters. In effect, he read off what we might now call instrumental-variable estimating equations (see Figure 1B). The model was heavily overidentified, there being some forty more estimating equations than unknown structural parameters. Wright's solution was a pragmatic one, in which parameter estimates were chosen to satisfy, as closely as possible, as many of the estimating equations as possible. This graphical approach, which he had first introduced in 1920, he referred to as "path analysis," the structural parameters being called "path coefficients" (see Figure 1C).

The Central System of Path Coefficients

	Hog breeding (B)		
a (Corn price)	−0.45	*b* (Summer price)	+0.35
c (Corn price′)	−0.85	*d* (Winter price′)	+0.65
e (Corn price″)	−0.15	*f* (Breeding′)	+0.10
	Summer price (S)		
g (Corn price)	+0.15	*i* (Breeding′)	−0.40
h (Corn price′)	+0.50	*j* (Breeding″)	−0.40
	Winter price (W)		
k (Corn price)	+0.45	*m* (Breeding′)	−0.65
l (Corn price′)	+0.25		

Figure 1C.—Wright (1925).

This corn-hog model is formidable, even by present-day standards. It is recursive, but it contains unobservable variables. How about simultaneity? In a footnote Wright (1925, p. 54) remarked that it would be desirable to specify a direct negative influence of hog quantity on hog price, that is, to introduce an explicit demand function. But "the treatment of such reciprocal relations between variables requires an extension of the theory of path coefficients." The extension was soon forthcoming. In 1928 Sewall Wright's father, the economist Phillip G. Wright, published *The Tariff on Animal and Vegetable Oils*. An appendix, prepared in collaboration with Sewall Wright, treats identifiability in supply-demand models as follows:

> The elasticities of supply and demand cannot be computed from price, output, and consumption data alone. The unknown quantities are too numerous for the equations .... Elasticity of supply (demand) can be computed only when assurance is obtained that the cost (demand) curve remains fixed while the demand (cost) curve is changing its position .... In the absence of [such] intimate knowledge ... statistical methods for imputing fixity to one of the curves while the other changes its position must be based on the introduction of additional factors ... which (A) affect demand conditions without affecting cost conditions or which (B) affect cost conditions without affecting demand conditions.

The Wrights take the static supply-demand model,

$$(3) \qquad q^D = \alpha p + u, \qquad q^S = \beta p + v, \qquad q^D = q^S \; (= q),$$

and observe z (the price of a substitute, or an index of prosperity) for which it is assumed that the covariance of z and v, $C(z, v)$, is zero:

$$C(z, v) = 0.$$

This implies that $C(z, q) = \beta C(z, p)$, so that $\beta = C(z, q)/C(z, p)$. Similarly, they observe x (yield per acre, or lagged price), assume $C(x, u) = 0$, and deduce $C(x, q) = \alpha C(x, p)$, from which $\alpha = C(x, q)/C(x, p)$. (I have modernized notation for present purposes.) They use this instrumental-variable procedure to produce estimates of the elasticities of demand and supply for butter (−.6, 1.4) and for flaxseed (−.8, 2.4).

This analysis, published in 1928, provides a much more general treatment of identification than Schultz offered in the same year, or indeed in 1938.

By 1934, Sewall Wright (then Professor of Zoology at the University of Chicago) had deepened his analysis of the supply-demand model. Examining the reduced form of (3), namely,

$$(4) \qquad q = (-\beta u + \alpha v)/(\alpha - \beta), \qquad p = (-u + v)/(\alpha - \beta),$$

he remarked that the three estimable moments, namely $V(q)$, $V(p)$, and $C(q, p)$, were insufficient to identify the five unknown parameters, namely α, β, $V(u)$, $V(v)$, and $C(u, v)$. (Again, I have modernized the notation.) He sketched the possible solutions: first we may observe z, a factor which shifts the demand curve, but not the supply curve. Then

$$(5) \qquad u = \gamma z + u' \qquad \text{where} \qquad C(z, v) = 0,$$

and the reduced form becomes

$$q = (-\beta\gamma z - \beta u' + \alpha v)/(\alpha - \beta), \qquad p = (-\gamma z - u' + v)/(\alpha - \beta).$$

There are now five estimable moments, namely $V(q)$, $V(p)$, $C(q, p)$, $C(z, q)$, $C(z, p)$, and six parameters, namely α, β, γ, $V(u')$, $V(v)$, and $C(u', v)$. Wright (1934, p. 199) then notes, "If it can safely be assumed that there is no correlation between the demand and supply situations [that is, if

$$(6) \qquad C(u', v) = 0],$$

a solution is possible." The five moments suffice to estimate the five unknown parameters. If such an assumption is not justified, he goes on, we may observe x, a factor which shifts the supply curve, but not the demand curve. Then

$$(7) \qquad v = \delta x + v' \qquad \text{where} \qquad C(x, u) = 0,$$

and the seven estimable moments, namely $V(q)$, $V(p)$, $C(q, p)$, $C(z, q)$, $C(z, p)$, $C(x, q)$, and $C(x, p)$, suffice to determine the seven parameters, namely α, β, γ, δ, $V(u')$, $V(v')$, and $C(u', v')$.

This analysis is quite remarkable and quite complete. It does miss the fact that each instrument suffices to identify one equation, and it does not touch on the overidentified case where both instruments are available *and* the disturbances are uncorrelated.

In the same article, Wright uses his method to estimate a supply-demand model for hogs. He also fits a supply-demand model for potatoes with lagged price as the instrument, checks the results against those obtained with lagged quantity as instrument, and develops a dynamic version of the model. He remarks, "I am indebted to my colleague, Professor Henry Schultz, for data on the quantity and price of potatoes marketed annually from 1896–1914 and the suggestion that it would be interesting material for analysis by this method" (p. 201).

It is reasonable to expect that, writing in 1938, Schultz would draw on Wright's work. Chapter 11 of Schultz (1938) is devoted to the demand for potatoes, and on p. 429 he lists nine previous studies; Wright's is not in the list. There are just four references to Sewall Wright in Schultz' book. On pp. 220 and 743 he credits Wright (1921, 1925, and 1934) for formulas allocating R^2 and for interpretation of beta-weights in multiple regression. On p. 739 he cites Wright (1934) as the source of the formula for the approximate variance of a nonlinear function, and on p. 275, along with seven other studies of the demand for corn, he lists, without comment, Wright's 1925 bulletin. Finally, on p. 73 he mentions, in a footnote, P. G. Wright's 1928 book for a discussion of the relation between theoretical and statistical supply and demand curves.

It is difficult to find an explanation for Schultz' failure to recognize, let alone utilize, Wright's powerful approach to the formulation and estimation of structural equation models. What makes the matter particularly puzzling is that Schultz and Wright were more than casually acquainted.

The physicist George Link (1942, p. 3) has left us a striking picture:

> [In 1933], Henry Schultz . . . heard Benjamin Willier, Sewall Wright, and myself refer to our hikes in the Indiana and Michigan dunes. He expressed his interest, was invited to accompany us, and promptly became a regular participant. Later Arthur Dempster and Samuel Allison became members of the group so that it included an embryologist, a geneticist, a pathologist, an economist, and two physicists. On every trip one or more of us carried a book, a journal, or a separate in the knapsack. It usually fell to my lot to read aloud while we sat around a fire, or in warm weather enjoyed either the sun or the shade. We had very spirited discussions on a wide range of old and contemporary subjects. Henry and Sewall shared special interests in mathematics and statistics, particularly the problems of probability. Henry also became interested in all sorts of things biological, including his own soma and

> psyche. In the winter Benjie, Sewall, Henry, and I often walked from Tremont to Michigan City and back along the beach where the sand and snow offered excellent facilities for execution of biologic diagrams, curves, and equations of all sorts.

Perhaps the only available explanation is that provided by Wright himself: "Henry was so committed to a completely formal objective approach in multivariate analysis that I could never get him to see the possible usefulness of my somewhat informal subjective mode of approach" (personal communication, September, 1971).

Schultz' heirs have hardly redressed the situation. I have been able to locate a bare handful of econometricians who have cited Wright's path analysis. The references by Koopmans and Reiersøl (1950, p. 167), Wold (1956, pp. 25 and 41; 1962; and 1965), Basmann (1960), and Fisk (1967, p. 1) are very brief. Only Karl Fox (1958, pp. 17–18) is more generous:

> I believe the work of Sewall Wright deserves increased attention. In his 1921 article "Correlation and Causation", Wright was . . . dealing with the problem of estimating causal relationships under circumstances in which it was impossible to eliminate other sources of variation in order to isolate the direct effects of one variable upon another . . . [H]is "method of path coefficients" may be considered a forerunner of the whole simultaneous equations approach . . . [In Wright (1934, p. 196)] he specifically addresses himself to the identification problem as it arises in connection with the economists' demand and supply functions. His equations on pp. 196–97 are strongly suggestive of the method of "reduced forms" which figures in the Cowles Commission literature, and his diagram and discussion on pages 198–99 point out that identification can be achieved only if, in addition to variables common to both equations, there is a variable in each that does not enter into the other. Henry Schultz [1938] took specific note of both of these articles by Wright but did not respond to them in any fundamental way.

For the record, it seems to me that Wright's (1928, 1934) treatment of identification and estimation was more comprehensive than that of two other econometric pioneers. H. Working (1925, pp. 526–539) focused only on the situations where price, or quantity, was in fact predetermined. E. J. Working (1927) focused on the relative stability of the demand and supply curves. He then remarked:

> Whether a demand or a supply curve is obtained may also be affected by the nature of the corrections [e.g., trend removal, deflation] applied to the original data. The corrections may be such as to reduce the effect of the shifting of the demand schedules without reducing the effect of the shifting of the supply schedules. In such a case the curve obtained will approximate a demand curve, even though the original demand schedules fluctuated fully as much as did the supply schedules. By intelligently applying proper refinements, and making corrections to eliminate separately those factors which cause demand curves to shift and those factors which cause supply curves to shift, it may be possible even to obtain both a demand curve and a supply curve . . . from the same original data.

This may be on the right track, but a large gap remains between E. J. Working's proposal to use exogenous shift variables as "corrections" (in an unspecified manner) and Sewall Wright's proposal to use them as instrumental variables (in a clearly specified manner).

3. WRIGHT'S PATH ANALYSIS

Wright's path analysis starts by displaying a structural model as a flow diagram with one-way arrows flowing from causal variables to their direct effects, and with curved two-headed arrows connecting exogenous variables (whose causal linkage is not investigated). The symbols on the one-headed arrows are path coefficients, i.e., structural coefficients. Estimation proceeds as follows, in the words of Wright (1960a, p. 193):

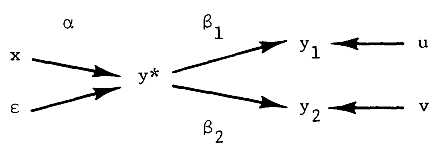
> The principle that was arrived at (for systems in which there are no paths that return on themselves) may be stated as follows: the correlations between any two variables in a properly constructed diagram of relations is equal to the sum of contributions pertaining to the paths by which one may trace from one to the other in the diagram without going back after going forward along an arrow and without passing through any variable twice in the same path. A coefficient pertaining to the whole path connecting two variables, and thus measuring the contribution of that path to the correlation, is known as a *compound path coefficient*. Its value is the product of the values of the coefficients pertaining to the elementary paths along its course. One, but not more than one of these, may pertain to a two-headed arrow without violating the rule against going back after going forward.

Remarkably enough, this "principle" leads one to write down valid equations relating population moments to parameters. When population moments are replaced by sample moments, one has a set of estimating equations of the type which econometricians will recognize from instrumental-variable and method-of-moments theory.

We illustrate with a trivial example taken from Wright (1960a). Let

$$(8) \qquad y^* = \alpha x + \varepsilon, \qquad y_1 = \beta_1 y^* + u, \qquad y_2 = \beta_2 y^* + v,$$

where x is exogenous; the disturbances ε, u, and v are mutually independent; y_1 and y_2 are endogenous; and y^* is an unobservable variable (standardized to have unit variance $\sigma_{**} = 1$). The path diagram is:



Independence among exogenous variables and disturbances is explicit in this diagram, since two-headed arrows are absent. Applying Wright's principle, we read off the following relations connecting parameters and population moments:

$$\sigma_{11} = \beta_1^2 + \sigma_{uu}, \qquad \sigma_{12} = \beta_1\beta_2, \qquad \sigma_{1x} = \alpha\beta_1\sigma_{xx},$$
$$(9) \qquad \sigma_{22} = \beta_2^2 + \sigma_{vv}, \qquad \sigma_{2x} = \alpha\beta_2\sigma_{xx},$$
$$1 = \alpha^2\sigma_{xx} + \sigma_{\varepsilon\varepsilon}.$$

These six equations suffice to identify the six structural parameters α, β_1, β_2, σ_{uu}, σ_{vv}, and $\sigma_{\varepsilon\varepsilon}$. When population moments are replaced by their sample counterparts, (9) gives us a set of perfectly respectable equations for estimating the parameters.

In Wright's hands, path analysis proved powerful and flexible enough to handle not only two-equation supply-demand systems, but also such topics as: bone sizes of rabbits (1918), skin color of guinea pigs (1920), birth weight of guinea pigs (1921), transpiration of plants (1921), inbreeding and assortative mating (1923), heredity, environment, and human intelligence (1931 and 1934), genetical structure of populations (1951), respiratory homeostasis (1921 and 1960b), and again birth weight of guinea pigs (1968). In these studies, he built up elaborate structural models from components as simple as those in our example. With few exceptions his models are recursive, and indeed the path analysis algorithm cannot be applied conveniently to the structural equations themselves when there is simultaneity (when "paths return on themselves"). When the models are overidentified—that is, when there are more estimating equations than unknown parameters—path analysis gives no systematic guide to efficient estimation. Wright's practice has been to average conflicting equations in an ad hoc manner, or to check informally that conflicting estimates are close together. Rarely does he spell out a rigorous statistical model, and he is often vague about the distinction between sample and population values. But as the following remark indicates, he was well aware of the gain in efficiency associated with overidentification: "... where there is a superfluity of equations for determining the path coefficients, the standard errors of these are correspondingly reduced" (1934, p. 210).

Statisticians have generally ignored Wright's work; exceptions are Tukey (1954), Kempthorne (1957), Moran (1961), and Dempster (1971, p. 338). (Moran's remark is suggestive: "The main reason why Sewall Wright's method of path coefficients is often found difficult to understand is that expositions of the theory do not make clear what assumptions are made.") But path analysis is well-known to population geneticists, who have applied it routinely in data analysis (for a lucid exposition, see Kempthorne (1957, ch. 14)). Sewall Wright has received numerous scientific awards for his contributions to genetics, and he is alive and well and working (at the age of 83) in Madison, Wisconsin.

Our explorer's chart is no doubt too crude for contemporary econometrics. Having rectified our scandalous neglect of his work, we might conclude that this work is of historical, not scientific, interest to us. But note that path analysis is quite capable of handling unobservable variables in structural equation models; many of Wright's studies do just that. Note, too, that Wright used overidentifying restrictions, albeit informally, to test his models; few macroeconometric model builders do any testing of restrictions. Most relevant for present purposes is the fact that it was his approach, rather than ours, which sparked the recent upsurge of causal modeling in sociology.

4. PATH ANALYSIS IN SOCIOLOGY

In 1966 Dudley Duncan published an exposition of path analysis for sociologists. Duncan illustrated the method with empirical models of family background, ambition, IQ, and class values; of city growth and population density; of occupational prestige ratings; and of intergenerational mobility among occupation and education brackets. Blau and Duncan (1967) published a comprehensive study of the American occupational structure, which includes a three-equation model of social stratification fitted by path analysis. All of these models were recursive and apart from conventional disturbances contained no unmeasured variables. Thus conventional regression analysis would have sufficed. Duncan (1969) built a causal model in which two unmeasured variables appear. Here the exogenous variables father's occupation, number of siblings, and ambition determine in a recursive manner the endogenous variables education, work orientation, initial job, and current job. Ambition and work orientation were unmeasured. Estimates, based on a sample of one thousand men, were developed by path analysis.

Since 1968, work by Duncan, his associates, and other sociologists have featured unmeasured variables—that is, theoretical constructs which are not directly observed, but which have implications for relationships among observables. Path analysis has been utilized to formulate and estimate fairly elaborate models of intergenerational mobility, educational achievement, motivation, occupational choice, teaching quality, attitude change, and even the use of drugs by college students. Overidentification is typically handled by various ad hoc averaging schemes rather than statistically efficient procedures. The distinction between sample and population is often obscured. Nevertheless, our neighbors have demonstrated a clear grasp of the idea that least-squares regression is an inadequate tool for structural estimation. They have also used overidentifying restrictions to "test" their models.

Initially they made sparse reference to the econometric literature. Simultaneity was introduced in a study of "peer influences," which concerns the way in which one's peers (e.g., best friends) influence one's decisions (e.g., choice of occupation). Duncan recognized that the relation must be reciprocal—if my best friend influences my choice, I influence his. Duncan, Haller, and Portes (1968) developed a simultaneous-equation model of peer influences on occupational choice, applying two-stage least squares to a sample of paired high-school students. They then reinterpret educational and occupational choice as two *indicators*—that is, erroneous measurements—of a single unmeasured "ambition" variable, and specify reciprocal causation between the friends' ambitions rather than between their choices. This yields the model of Figure 2, which combines simultaneity and

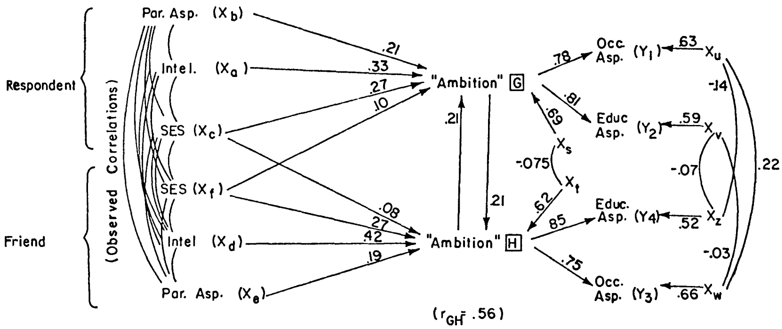


FIGURE 2.—Duncan, Haller, and Portes (1968).

errors in the variables. With no clear-cut econometric approach available, estimates were prepared by path analysis.

Currently, path diagrams and structural models fill the pages of the sociological journals and have begun to crop up in political science. Educational psychologists were introduced to path analysis by Werts and Linn (1970), and the economist H. Gintis (1971) reported on a path analysis of education and worker productivity. Griliches and Mason (1972) employ some path analysis in a study of education and income, treating aptitude tests as indicators of an unobservable ability variable. Hauser and Goldberger (1971) cast some path analyses into a formal stochastic framework. Blalock (1971) provides a good anthology of the literature of several disciplines.

5. FACTOR ANALYSIS IN PSYCHOLOGY

In contrast to sociologists, psychologists have a long history of work with structural models. In particular, educational psychologists have used factor analysis widely since the 1930's. A prototype example will say that a student's scores on a battery of tests are determined by a small number of unobservable mental abilities.

The basic factor analysis model specifies a set of linear relationships in which M observable variables (indicators) $y_1, \ldots, y_M$ are determined by K unobservable variables (common factors) $z_1, \ldots, z_K$, and M independent disturbances (unique factors) $u_1, \ldots, u_M$. In matrix terms, the model has

$$(10) \qquad y = \Lambda z + u, \qquad E(uu') = \Theta \text{ (diagonal)}, \qquad E(zu') = 0,$$

where Λ is the $M \times K$ matrix of factor loadings (structural coefficients). This implies that the covariance matrix of the observables is

$$(11) \qquad \Sigma = E(yy') = \Lambda \Phi \Lambda' + \Theta, \qquad \text{where } \Phi = E(zz'),$$

and has empirical content insofar as it imposes constraints on Σ. To display this, normalization rules may be introduced, say

$$\Phi = I, \qquad \Lambda'\Theta^{-1}\Lambda = \Delta \text{ (diagonal)}.$$

It is then apparent that the moments among the observable variables, namely the $q = M(M + 1)/2$ distinct elements of Σ, are expressed in terms of

$p =$	MK	(elements of Λ)
	$+ M$	(nonzero elements of Θ)
	$+ K$	(nonzero elements of Δ)
	$- K(K + 1)/2$	(distinct constraints in $\Lambda'\Theta^{-1}\Lambda = \Delta$)
	$= M(K + 1) - K(K - 1)/2$	

free parameters. This leaves

$$r = q - p = ((M - K)^2 - (M + K))/2$$

restrictions on Σ, a quantity which is positive when M is substantial relative to K. For example, if $M = 10$ test scores are expressed in terms of $K = 3$ abilities, there are $r = 55 - 37 = 18$ restrictions on Σ.

These restrictions on Σ resemble the restrictions on the reduced-form coefficient matrix Π which arise in overidentified simultaneous-equation models (see Koopmans and Reiersøl (1950) and Goldberger (1971b)). In factor analysis the sample covariance matrix of y provides an unconstrained estimate of Σ, just as in simultaneous-equation models the multivariate regression of endogenous on exogenous variables provides an unconstrained estimate of Π. Efficient parameter estimation is a matter of reconciling the conflicting unconstrained estimates. In factor analysis, as in econometrics, a large number of estimation methods have been developed. The list includes the counterparts of our full-information maximum-likelihood, simultaneous least squares, and minimum-distance (with diagonal distance matrix) methods. A new addition to the list is the counterpart of Zellner-Aitken estimation, i.e., minimum-distance with distance matrix developed from unconstrained sample estimates (see Jöreskog and Goldberger (1972)). It is no coincidence that identification rules and estimation procedures for factor analysis were worked out by Anderson and Rubin (1956), whose contributions to simultaneous-equation theory are well-known to econometricians.

After the factor model has been fitted, the normalization rules can be removed by a sequence of ex post transformations which attempt to reveal a simpler parametric structure. Transformed factors, $z^* = Az$, are sought such that

$$\Lambda^* = \Lambda A^{-1} \quad \text{and} \quad \Phi^* = E(z^*z^{*\prime}) = AA' \tag{12}$$

have neat patterns. The idea is to make the common-factor portion of Σ, namely

$$\Lambda\Lambda' = \Lambda^*\Phi^*\Lambda^{*\prime}, \tag{13}$$

more readily interpretable. Concern with this rotation phase, which has no direct econometric counterpart, occupies a good part of many factor analysis texts, contributing to their unreadability. The book by Lawley and Maxwell (1971) is a noteworthy exception.

In view of the many parallels, it is strange that econometricians have paid little attention to the factor analysis literature. I suspect that we confuse factor analysis with principal component analysis, and view the latter as a mechanical procedure for reducing dimensionality in regression computations. Some of us have read Malinvaud (1970, pp. 410–411) or Anderson (1963) or Madansky (1964) and realize that factor analysis is just a generalization of the classical errors-in-the-variables model; but we associate errors in the variables with underidentification, an unattractive property. Even those who realize identification is quite possible when errors of measurement are present have shown no enthusiasm for factor analysis. Two explanations for the lack of enthusiasm come to mind: (i) economists are not attracted by models in which variables and parameters are redefined ex post, and (ii) economists are not attracted by models in which *all* observable variables are treated symmetrically as effects of unobservable causes.

These features of traditional factor analysis would indeed make it inappropriate for economists, who draw on well-defined variables and well-defined chains of causality. But these features are not inherent in factor analysis. It is possible to build a priori specifications, e.g., zeroes and equalities, into the parameter matrices. These are quite parallel to the zero restrictions in the structural equations of our simultaneous-equation models. When enough such assumptions are built in, ex post transformation of variables and parameters is ruled out. It is quite possible to work with higher-order factor analysis, in which a causal structure is imposed among the factors, so that factor covariances are derived, rather than primary, parameters. For these features of confirmatory factor analysis, see Lawley and Maxwell (1971, ch. 6), Jöreskog (1969, 1970), and Anderson and Rubin (1956, secs. 5, 7, 10).

Jöreskog (1970) extends the factor analysis model to a situation where the covariance matrix of the observable variables y takes the form

$$\Sigma = B(\Lambda\Phi\Lambda' + \Theta)B' + \Psi, \tag{14}$$

where B, Λ, Φ (symmetric), Θ (diagonal), and Ψ (diagonal) are parameter matrices which may be subject to various types of linear constraints. Furthermore, the expectation of y is permitted to vary linearly with observable exogenous variables, and the coefficients in these regressions may be subjected to linear constraints as well. As Jöreskog shows, this scheme is rich enough to encompass a wide range of the structural models which have concerned psychologists dealing with test scores, higher-order factor analysis, multivariate analysis of variance, multi-test multi-occasion data, components of variance, growth curves, Weiner stochastic processes, and even some path analyses. The maximum-likelihood algorithm is routinized for Jöreskog's general covariance structure. As it turns out, the general structure is not quite general enough to cover econometric simultaneous-equation models, but Jöreskog (1973) has since worked out an adaptation of his approach which allows for errors in the variables as well as simultaneity, and has illustrated the method with a macroeconomic model. He now occupies the chair left vacant by Herman Wold's departure from the University of Uppsala. The fences which have divided the fields of econometrics and psychometrics are falling down.

This concludes our visit with structural equation model builders in the other social sciences. The main message to take away is that it is feasible, and often desirable, to handle unobservable variables in the context of multi-equation structural models. The work of Wright, Duncan, and Jöreskog demonstrates this.

6. UNOBSERVABLE VARIABLES IN ECONOMICS

Unobservable variables, or errors of measurement, or errors in the variables have a curious history in econometric theory. In the early days, economic equations were formulated as exact relationships among unobservable variables, and errors in the variables provided the only stochastic component in the observations. But, at least since the days of the Cowles Commission, the emphasis has shifted almost entirely to errors in the equations. (Distributed lag models provide an exception: there "anticipated" and "desired" values are unobservables). Today, many econometrics textbooks offer only a very casual treatment of errors in the variables. The official justification is that in economic data, measurement errors are negligible, at least in comparison with behavioral disturbances.

The real explanation may lie elsewhere. Economists have taken the permanent income model as their prototype of the errors-in-the-variables set-up. We know that this model is underidentified (at least without Friedman's special assumption that permanent consumption is proportional to, rather than merely linear in, permanent income). Consequently we have come to associate errors in the variables with underidentification. Since underidentified models present no interesting problems of estimation and testing, econometric theorists have turned away. Several econometrics texts give the impression that a combination of measurement error and simultaneity is intractable: see Johnston (1963, p. 294), Goldberger (1964, pp. 387–388), and Kmenta (1971, pp. 321–322, 596).

Neglect by theorists evokes malpractice by empiricists. In empirical econometrics it is not uncommon to find "proxy" or "surrogate" variables used freely, with little effort made to trace out the consequences. Sometimes measurement error comes in as a last resort: if, after much experimentation with the choice of explanatory variables in a regression, I am still unable to get the right results, I remark "of course, the variables must have been inaccurately measured." When nothing is possible in the way of rigorous model building, identification, estimation, and testing, then everything is possible and anything goes in the way of rationalization.

However, the permanent income model is a very misleading prototype, because for its single unobservable variable (permanent income) we observe only two indicators (income and consumption). More precisely, the model is

$$(15) \qquad y_1 = \beta_1 y^* + u, \qquad y_2 = \beta_2 y^* + v; \qquad y^*, u, \text{ and } v \text{ independent.}$$

Here y_1 is income, y_2 is consumption, y^* is permanent income, and the usual $\beta_1 = 1$ will be replaced by the normalization $\sigma_{**} = V(y^*) = 1$, which is more convenient. The model implies three equations connecting moments of the observable variables with structural parameters, namely

$$\sigma_{11} = \beta_1^2 + \sigma_{uu}, \qquad \sigma_{12} = \beta_1\beta_2, \qquad \sigma_{22} = \beta_2^2 + \sigma_{vv}. \tag{16}$$

These three equations do not suffice to determine the four parameters β_1, β_2, σ_{uu}, and σ_{vv}. The model is indeed underidentified.

But now suppose that we observed three, rather than two, indicators of permanent income:

$$y_1 = \beta_1 y^* + u, \qquad y_2 = \beta_2 y^* + v, \qquad y_3 = \beta_3 y^* + w; \qquad y^*, u, v, \text{ and } w \text{ independent.} \tag{17}$$

This in conjunction with $\sigma_{**} = 1$ would imply

$$\begin{aligned} \sigma_{11} &= \beta_1^2 + \sigma_{uu}, & \sigma_{12} &= \beta_1\beta_2, & \sigma_{13} &= \beta_1\beta_3, \\ & & \sigma_{22} &= \beta_2^2 + \sigma_{vv}, & \sigma_{23} &= \beta_2\beta_3, \\ & & & & \sigma_{33} &= \beta_3^2 + \sigma_{ww}. \end{aligned} \tag{18}$$

These six equations just suffice to determine the six parameters β_1, β_2, β_3, σ_{uu}, σ_{vv}, and σ_{ww}. The model is now just-identified and readily estimable.

If a fourth indicator of the same type were available, the model would be overidentified, thus not only estimable, but actually testable. We see that *multiple indicators* provide one route to identifiability when unobservable variables are present. This is the route followed by factor analysis.

Another route, which is likely to be more appealing to economists, was pointed out by Zellner (1970), who relies on *multiple causes* rather than multiple indicators. His model is simply

$$y_1 = \beta_1 y^* + u, \qquad y_2 = \beta_2 y^* + v, \qquad y^* = \alpha_1 x_1 + \cdots + \alpha_K x_K; \qquad u, v \text{ mutually independent and independent of } x_1, \ldots, x_K. \tag{19}$$

Here y_1 and y_2 are the observable indicators of the unobservable variable y^*, which is an exact linear function of the observable exogenous variables $x_1, \ldots, x_K$. Solving for each indicator in terms of the causes, Zellner finds the reduced form:

$$\begin{aligned} y_1 &= \beta_1\alpha_1 x_1 + \cdots + \beta_1\alpha_K x_K + u, \\ y_2 &= \beta_2\alpha_1 x_1 + \cdots + \beta_2\alpha_K x_K + v, \end{aligned} \tag{20}$$

and notes that it is overidentified when $K > 1$. There are $K - 1$ constraints resulting from the fact that the $2K$ reduced-form coefficients are expressed in terms of $K + 1$ free parameters—K α's, 2 β's, less one normalization. (Zellner's normalization is $\beta_1 = 1$; an alternative is $\sigma_{**} = 1$.)

In Zellner's model, the $K \times 2$ reduced-form coefficient matrix Π has rank 1. This will be recognized as a special case of the pattern which arises in limited-information estimation of a single equation in a conventional simultaneous-equation model. The added twist here is that the reduced-form disturbances are uncorrelated, a specification which is exploited by Zellner in working out the Zellner-Aitken estimation algorithm for his model. Goldberger (1972) develops the full-information maximum-likelihood algorithm for the same model, and then introduces a disturbance ε into the equation determining y^*. In the present two-indicator case, this is essentially equivalent to permitting u and v to be correlated. Consequently, Zellner-Aitken estimation produces estimates identical with FIML, according to a theorem given in Goldberger and Olkin (1971).

All this falls neatly within mainstream econometrics, since the overidentifying restrictions are just a special case of those which arise in simultaneous-equation models. But now suppose that we have more than two, say M, observable indicators, with independent errors. In matrix notation the model is written

$$\begin{aligned} \underset{(M \times 1)}{y} &= \underset{(M \times 1)}{\beta} \; \underset{(1 \times 1)}{y^*} + \underset{(M \times 1)}{u} \\ \underset{(1 \times 1)}{y^*} &= \underset{(1 \times K)}{\alpha'} \; \underset{(K \times 1)}{x} + \underset{(1 \times 1)}{\varepsilon}, \end{aligned} \tag{21}$$

$$E(uu') = \Theta \text{ (diagonal)}, \qquad E(xu') = 0, \qquad E(x\varepsilon) = 0, \qquad E(u\varepsilon) = 0.$$

Normalizing by $\sigma_{\varepsilon\varepsilon} = 1$, and passing to the reduced form, we have

$$\begin{aligned} y &= \beta\alpha'x + (\beta\varepsilon + u) = \Pi'x + v, \\ \Omega &= E(vv') = \beta\beta' + \Theta. \end{aligned} \tag{22}$$

This multiple-indicator, multiple-cause model offers a mixture of econometric and psychometric themes. The fact that $\Pi = \alpha\beta'$ means that the reduced-form coefficient matrix has rank 1; this is again of the type which arises in simultaneous-equation models. The fact that $\Omega = \beta\beta' + \Theta$ with Θ diagonal means that the reduced-form disturbance covariance matrix is of the type which arises in factor analysis (ε plays the role of the single common factor and u the role of the unique factors). Thus, the reduced-form coefficients have an econometric pattern, while the reduced-form disturbances have a factor-analytic pattern. Furthermore—and this is what makes the model particularly intriguing—the factor-loading vector β which enters into Ω coincides with the structural-coefficient vector β which enters Π. A factor-analytic pattern on disturbances arises in the variance-components model developed by Balestra and Nerlove (1966) for use on panel data, and a connection between regression coefficients and disturbance covariances arises in some of Theil's (1967, pp. 228–233) consumer demand models, but the present setting is distinctive.

A FIML algorithm is developed by Jöreskog and Goldberger (1973). Concentrating the likelihood function, we observe that, conditional on Θ, the problem reduces to finding the scalar μ^2 which minimizes

$$f(\mu^2) = \log(1 + \mu^2) - \lambda(\mu^2). \tag{23}$$

Here $\lambda(\mu^2)$ is the largest characteristic root of the matrix

$$A = \frac{\mu^2}{1 + \mu^2} S + Q, \tag{24}$$

while S and Q are respectively the residual moment matrix and the regression moment matrix in an unconstrained multivariate regression of transformed y variables on the x variables. Note how the maximum-likelihood procedure mixes information coming from the regression coefficients with information coming from the residuals. This corresponds to the fact that the structural parameter vector β enters both Π and Ω. If β entered Π only, then $\mu = 0$, and the problem would reduce to finding the largest root of Q, as in the familiar LIML algorithm.

If the multiple-indicator, multiple-cause model is to be useful for economists, it will, most likely, have to be embedded in larger systems. The larger systems would have several unobservable variables y^*, whose indicators y determine other endogenous variables, and whose causes x are themselves determined by other exogenous variables. This has already occurred in some sociological path models. Efficient estimation of these larger systems remains an open field, which econometric theorists might wish to investigate.

Now, our multiple-indicator, multiple-cause scheme is strictly recursive. To round out the discussion, we should consider the possibility of allowing for unobservable variables within simultaneous-equation models. The first point is an obvious one, namely that random errors of measurement in the endogenous variables do not affect identifiability of the structural coefficients. The second point is less obvious, but is implicit in a passage in Chernoff and Rubin (1953, pp. 204–206) which later writers—except for Sargan (1958)—seem to have overlooked. Errors of measurement in exogenous variables need not destroy identifiability, provided that the model is otherwise overidentified. In effect, one can trade off overidentifying restrictions against the underidentifiability introduced by measurement error.

Consider a conventional simultaneous equation model with reduced form

$$y' = x^{*\prime}\Pi^* + v', \qquad E(x^*v') = 0. \tag{25}$$

We suppose that the model is overidentified, so that not only are the parameters of the structural equations identified in terms of Π^*, but there are constraints on Π^*. Now suppose that x^* is unobservable, but that we observe x which is related to x^* in the conventional errors-in-variables manner:

$$x = x^* + \varepsilon, \qquad E(x^*\varepsilon') = 0, \qquad E(v\varepsilon') = 0. \qquad E(\varepsilon\varepsilon') = \Theta \text{ (diagonal)}. \tag{26}$$

We suppose that Θ is not only diagonal, but also has some zeroes on the diagonal, reflecting the accurate measurement of some exogenous variables. Let $\Phi = E(xx')$

be the population covariance matrix of the observed x, and let Π be the coefficient matrix in the population linear regression of y on the observed x. Then

$$\Phi = E(xx') = E((x^* + \varepsilon)(x^{*\prime} + \varepsilon')) = E(x^*x^{*\prime}) + \Theta, \tag{27}$$

and

$$\begin{aligned}\Phi\Pi &= E(xy') = E((x^* + \varepsilon)(x^{*\prime}\Pi^* + v')) = E(x^*x^{*\prime})\Pi^* \\ &= (\Phi - \Theta)\Pi^*.\end{aligned} \tag{28}$$

Thus we find

$$\Pi^* = (\Phi - \Theta)^{-1}\Phi\Pi, \tag{29}$$

which relates the estimable moments and coefficients Φ and Π to the parameters Π^* and Θ. In conjunction with the overidentifying restrictions on Π^*, and the zeroes in Θ, this relation *may* suffice to identify Π^*, and consequently to identify the parameters of the structural equations.

To illustrate, consider Tintner's meat market model:

$$\begin{aligned} y_1 &= a_1y_2 + a_2x_1^* &&+ u_1 \quad \text{(demand)},\\ y_1 &= b_1y_2 &&+ b_2x_2^* + b_3x_3^* + u_2 \quad \text{(supply)},\end{aligned} \tag{30}$$

where the endogenous variables are y_1 (meat consumption) and y_2 (meat price), and the exogenous variables are x_1^* (income), x_2^* (wage rate), and x_3^* (materials price). The reduced form of the model is

$$(y_1, y_2) = (x_1^*, x_2^*, x_3^*)\begin{pmatrix}\pi_{11}^* & \pi_{12}^* \\ \pi_{21}^* & \pi_{22}^* \\ \pi_{31}^* & \pi_{32}^*\end{pmatrix} + (v_1, v_2). \tag{31}$$

The reduced-form coefficients are constrained by the fact that π_{21}^*/π_{22}^* and π_{31}^*/π_{32}^* are both equal to a_1, thus

$$\begin{vmatrix}\pi_{21}^* & \pi_{22}^* \\ \pi_{31}^* & \pi_{32}^*\end{vmatrix} = 0. \tag{32}$$

We suppose that there is a random error of measurement in x_1^*, but none in x_2^* nor x_3^*. Thus we observe the exogenous variables

$$x_1 = x_1^* + \varepsilon_1, \qquad x_2 = x_2^*, \qquad x_3 = x_3^*, \tag{33}$$

and the measurement-error covariance matrix takes the form

$$\Theta = \begin{pmatrix}\theta & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0\end{pmatrix}, \tag{34}$$

where $\theta = V(\varepsilon_1)$. Applying (29), we find

$$(35) \qquad \begin{pmatrix} \pi^*_{11} & \pi^*_{12} \\ \pi^*_{21} & \pi^*_{22} \\ \pi^*_{31} & \pi^*_{32} \end{pmatrix} = \frac{1}{1-\theta\phi^{11}} \begin{pmatrix} 1 & 0 & 0 \\ \theta\phi^{21} & 1-\theta\phi^{11} & 0 \\ \theta\phi^{31} & 0 & 1-\theta\phi^{11} \end{pmatrix} \begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \\ \pi_{31} & \pi_{32} \end{pmatrix},$$

where the ϕ^{ij} denote elements of Φ^{-1}; then use of (32) yields

$$(36) \qquad \frac{1}{1-\theta\phi^{11}} \begin{vmatrix} \theta\phi^{21}\pi_{11} + (1-\theta\phi^{11})\pi_{21} & \theta\phi^{21}\pi_{12} + (1-\theta\phi^{11})\pi_{22} \\ \theta\phi^{31}\pi_{11} + (1-\theta\phi^{11})\pi_{31} & \theta\phi^{31}\pi_{12} + (1-\theta\phi^{11})\pi_{32} \end{vmatrix} = 0,$$

which boils down to

$$(37) \qquad \theta = (\pi_{22}\pi_{31} - \pi_{21}\pi_{32})/[\phi^{11}(\pi_{22}\pi_{31} - \pi_{21}\pi_{32}) + \phi^{21}(\pi_{11}\pi_{32} - \pi_{31}\pi_{12}) + \phi^{31}(\pi_{21}\pi_{12} - \pi_{11}\pi_{22})].$$

Recalling that the ϕ's and π's are estimable from observations on y and x, we see that (37) suffices to identify θ, whence (35) suffices to identify the π^*'s, which in turn identify the a's and b's. The model, in fact, is now just-identified, the single overidentifying restriction having been traded off against the single measurement error variance.

This approach was applied to Tintner's model by Goldberger (1971a) using annual data 1919–41. The resulting reduced-form and structural coefficient estimates are reported below, along with the FIML estimates which result when all variables are assumed to be accurately measured (see Goldberger (1964, pp. 338–345)). The estimated error variance in income, $\theta = 551$, represents about 15 per cent of the total variance of income.

Reduced-Form Estimates

Π^*, with measurement error

$$\begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{cc} y_1 & y_2 \\ \left(\begin{array}{rr} -.059 & .183 \\ -.341 & .200 \\ .742 & -.435 \end{array}\right) \end{array},$$

Π^*, without measurement error

$$\begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{cc} y_1 & y_2 \\ \left(\begin{array}{rr} -.049 & .119 \\ -.215 & .044 \\ .701 & -.145 \end{array}\right) \end{array}.$$

Structural Estimates

	a_1	a_2	b_1	b_2	b_3	θ
With measurement error	−1.71	.20	−.32	−.28	.60	551
Without measurement error	−4.85	.53	−.41	−.20	.64	—

Our example indicates that it is possible to handle errors of measurement in the context of simultaneous-equation models. It also indicates problems which remain to be resolved. What are the rules for identification in the general case of a simultaneous-equation model with unobservable variables? (Not only the number, but also the location of the measurement errors, is crucial.) Anderson and Hurwicz (1949) sketched out the subject; further results on identification were obtained by Wiley (1973) and by Geraci and Goldberger (1971); but this remains an open territory for econometric theorists.

7. CONCLUSION

Following Wold, we have seen how other social scientists share our concern with the methodological problems associated with structural equation models. To conclude, we should also follow Wold (1969, pp. 377–381) in remarking that other social scientists share our concern with substantive problems for which structural equation models are needed. For example, economic, sociological, psychological, and political theory all have something to say about the causal links among family background, education, and income which bear on the problem of poverty. This second community of concern provides an added incentive for econometricians to break through those fences which still separate the social sciences.

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